Homework 2 solutions

① ac Conductivity in a metal

Kinetic equation:

$$\frac{\partial f}{\partial t} + \cancel{\vec{v}_{\vec p}\frac{\partial f}{\partial \vec r}} + e\vec{E}(t)\frac{\partial f}{\partial \vec p} = -\frac{f-f_0}{\tau}$$

$\cancel{\vec{v}_{\vec p}\frac{\partial f}{\partial \vec r}} \to 0$ we are looking for a uniform distribution

Oscillating field: $\vec{E}(t) = \vec{\mathcal{E}}\, e^{-i\omega t}$

look for an oscillating solution $f - f_0 = \tilde{f}_1 e^{-i\omega t}$

In the rhs $e\vec{\mathcal{E}}\frac{\partial f}{\partial \vec p} \approx e\vec{\mathcal{E}}\frac{\partial f_0}{\partial \vec p} = e\vec{\mathcal{E}}\vec{v}\,\frac{\partial f_0}{\partial \varepsilon}$

Then

$$-i\omega\,\tilde{f}_1 + e\vec{\mathcal{E}}\vec{v}\,\frac{\partial f_0}{\partial \varepsilon} = -\frac{1}{\tau}\tilde{f}_1$$

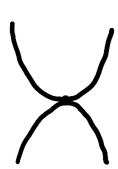
$$\rightarrow\quad \tilde{f}_1 = -e\vec{\mathcal{E}}\vec{v}\,\frac{\partial f_0}{\partial \varepsilon}\,\frac{1}{1-i\omega\tau}$$

$$\rightarrow\quad f - f_0 = -\frac{1}{1-i\omega\tau}\, e\vec{\mathcal{E}}\vec{v}\,\frac{\partial f_0}{\partial \varepsilon}$$

From here proceed exactly like with the dc conductivity

$$\boxed{\sigma(\omega) = \frac{\sigma_0}{1-i\omega\tau}}$$

Bonus

①  $\varepsilon_k = \pm\frac{k^2}{2m}$

We consider just one valley in bilayer graphene = one point in momentum space where the bands touch (in fact, there are several)

... the same way

touch (in fact, there are several,

<u>DoS</u>: (found in exactly the same way as in a metal)

Pick some energy ε, for example, in the conduction band (this choice will not affect anything). It corresponds to some momentum k in that band.

$$\frac{2\pi k\, dk}{(2\pi)^2} = \nu(\varepsilon)\, d\varepsilon \rightarrow \nu(\varepsilon) = \frac{k}{2\pi}\frac{dk}{d\varepsilon} = \frac{m}{2\pi}$$

Here we disregard spin and valley degeneracy; the expression is per unit area

Total energy of excitations

$$E = 2\int_0^\infty \nu(\varepsilon) f(\varepsilon)\, \varepsilon\, d\varepsilon$$

because there are 2 symmetric bands

$$f(\varepsilon) = \frac{1}{e^{\frac{\varepsilon}{T}}+1}$$

$$E = 2\int_0^\infty \frac{m}{2\pi}\frac{\varepsilon\, d\varepsilon}{e^{\frac{\varepsilon}{T}}+1} = \frac{mT^2}{\pi}\underbrace{\int_0^\infty \frac{dx}{e^x+1}}_{=\frac{\pi^2}{12}}$$

$$E = \frac{\pi m}{12} T^2$$

$$\rightarrow \boxed{C = \frac{\pi m}{6} T}$$ per spin per valley per unit area

②

$$C = \frac{\pi^2}{3} T\, \nu(\varepsilon_F)$$

Density of states (DoS) at the Fermi surface

See Abrikosov, Chapter 2